Department of Mathematics
C138 Padelford
Box 354350
University of Washington
Seattle, WA 98195-4350

206 543-1150 phone
206 543-0397 fax
www.math.washington.edu

DEPARTMENT OF MATHEMATICS
College of Arts & Sciences
UNIVERSITY OF WASHINGTON

John M. Lee
206 543-1735
lee@math.washington.edu

March 4, 2009

Anna-Maria de la Fuente
Seattle Public Schools
MS 32-156
PO Box 34165
Seattle, WA 98124-1165

Dear Anna-Maria,

I had a chance to look over some of the high-school math textbooks that are being considered for adoption, and I would like to pass along some of my thoughts and impressions to you, to the adoption committee, and to the School Board.

First, let me say a few words about my background. In addition to being a parent of two Seattle Public School students (one current and one former), I'm a professor of mathematics at UW, with 26 years experience doing mathematical research and teaching at the college and graduate levels. For the past 2½ years, I've been teaching the *Geometry for Teachers* course offered by our department, which introduces undergraduate and Master's students who are hoping to become high-school math teachers to the rigorous mathematical theory underlying geometry. I also have a bit of experience teaching at the middle- and high-school levels: Before going to grad school, I taught math, physics, and computer science to 7th–12th graders for two years at Wooster School (an independent school in Connecticut); and in 2003–2004, I created and taught an after-school math enrichment class (algebra and geometry) for 8th graders at TOPS school in Seattle—a class that was so popular that it has become self-sustaining, and typically enrolls 35%–40% of TOPS 8th graders each year.

Because the adoption committee's process has been pretty thorough about ensuring that the content of all of the candidate textbooks aligns well with the state standards, I didn't worry much about precisely what is covered in the different books; instead, I focused my attention primarily on how effectively the material is developed and presented. I was looking particularly for two indicators of quality: (#1) how well the book introduces new concepts by tying them in with ideas students already know, describing examples that illustrate both concepts and techniques, suggesting appropriate inquiry and discovery activities, and providing teacher support for explanations and in-class activities; and (#2) how well the book, after completing the inquiry/discovery portion of a lesson, gives clear and unambiguous statements of the mathematical conclusions that students need to draw from the lesson, especially definitions, postulates, theorems, and proofs in geometry, and computational algorithms, problem-solving techniques, and formulas in algebra.

That last point might deserve a bit of expansion, so please forgive me for getting on my soapbox for a moment. Mathematics is unique among fields of human endeavor in the level of certainty with which we can believe its conclusions. We know the ratio of the circumference of a circle to its diameter to be *exactly* π, and we can calculate it to far greater accuracy than we will ever be able to measure. No other field of scientific or humanistic study is ever able to claim this degree of certainty, but the certainties of mathematics provide critical underpinnings for the construction of models in virtually every natural and social science. (Of course, no human understanding is 100% certain, because there might be subtle mistakes somewhere in our logic that we haven't caught yet; but relative to other fields of understanding, mathematical knowledge is vastly more certain.)

The reason we are able to be so certain is that mathematics is a *deductive discipline*. Although our discovery and intuitive understanding of most mathematical truths are based on inductive and empirical reasoning from real-world experience, ultimately the certainty of our *knowledge* of mathematics is founded on logical deduction. The dazzling array of products of mathematical deduction constitutes one of the crowning achievements of the human mind. Therefore, a course that does not imbue students with the spirit, techniques, and practice of deductive reasoning is not a mathematics course worthy of the name. In geometry courses, this means a substantial introduction to definitions, postulates, theorems, and proofs. In algebra, it means taking patterns discerned from looking at examples, and translating them into computational rules, algorithms, and problem-solving techniques that can be applied efficiently and in great generality.

OK, now I'll climb down off my soapbox and tell you what I thought of the individual texts.

Geometry Texts

Because of my recent experience teaching *Geometry for Teachers*, I spent most of my time studying the three geometry texts. As far as I could tell, they all seemed to meet my criterion #1 (introducing new concepts) admirably, presenting an exciting array of inquiry activities for students and their teachers to use in gaining insight into geometric truths. So I'll focus my comments mostly on my impressions of how they met criterion #2 (clear statements of mathematical conclusions). This criterion is perhaps more important for geometry texts than anywhere else in the curriculum, because high-school geometry is the *only* place where most people are ever introduced to serious deductive reasoning.

Prentice-Hall: Geometry

I think this is a very good geometry text. It's remarkable for managing to achieve a fine balance of inductive and deductive reasoning, hands-on activities and clear explanations. It starts with a nice informal introduction to the "tools of geometry," which introduces such things as compass-and-straightedge constructions, ruler and protractor measurements, paper folding, applications of geometry to the real world, and the beginnings of deductive reasoning in the form of some basic postulates for geometry. This is an excellent way to get students engaged, and it should make it clear to them from the beginning that geometry is something that can easily be related to their everyday experience.

After this introduction, right away in Chapter 2 the book begins to delve deeply into deductive reasoning. Students are offered a thorough grounding in the laws of logic and the ideas of postulates and theorems, with examples of how deductive reasoning works in both algebra and geometry. Most importantly, there are clear examples of well-written proofs in various formats (two-column, paragraph-style, and flowchart), which students can learn from and emulate.

After that, the development proceeds in a well-organized and logical fashion. New concepts are introduced first through activities and examples that make the ideas come alive; then the students are encouraged to conjecture general conclusions from their inquiry experiences; and finally, the general principles are clearly stated as theorems, and *they are proved*, either by presenting a complete proof in the book, or by guiding students to create a proof on their own with appropriate suggestions and hints where needed. The descriptions and proofs are concise and easy to read, without being telegraphic.

The presentation is graphically extremely effective. All new mathematical terms are both boldfaced and highlighted in yellow when they are first defined. All mathematical statements are displayed in colorcoded boxes, showing clearly which statements are definitions, which are postulates, and which are theorems. At the end of the book, there is a table of all the postulates and theorems, with page references to where they are introduced and proved, and an amazingly comprehensive glossary of mathematical terms (in both English and Spanish!), again with page references. The choices of terminology, mathematical definitions, theorem statements, and proofs are mathematically sound throughout the book.

One thing that impressed me about the book was, paradoxically, the things the authors chose *not* to prove. Starting in Chapter 9 (Transformations), most theorems are presented without proofs. The textbook itself doesn't say why, but the explanation is given in the teacher's manual: "the proofs [of the theorems in this chapter] are too complicated for students at this point." Knowing what's involved in proving all of those theorems, I think this is a sound judgment. And even where proofs are eschewed, each new mathematical fact is presented as a clearly stated theorem, so that students (who by this time should be well acquainted with how proof leads to certain knowledge) will understand clearly that these results are on the same logical footing as all the previous theorems in the book, and not just empirical conclusions from experience.

Key Curriculum Press: Discovering Geometry

There is a lot to like about this book. I was impressed with a number of things about the presentation: the incorporation of art, both to decorate the text and to motivate geometric ideas; the use of color, typography, and boxes to indicate the functions of different parts of the text; the cartoons. The inquiry activities seem appropriate and engaging; the explanations of mathematical definitions and conclusions are sound; and the exercises give students an excellent array of opportunities to develop skill in thinking about all aspects of the subject.

However, this book had one serious drawback that makes it, in my opinion, vastly inferior to the Prentice-Hall text. For some reason, the authors have chosen to put off a serious treatment of proofs until Chapter 13, the very last chapter of the book. To their credit, they figured out an intellectually honest way to do this: Every mathematical statement in the book prior to Chapter 13 is stated as a *Conjecture*, so students will not be misled into thinking that the general principles they have arrived at by inductive reasoning from examples have the same logical status as theorems. There are some examples of proofs of these conjectures in the earlier chapters, but since a detailed axiomatic foundation has not yet been presented, the authors are careful to say that all these proofs show is that *if* conjectures X and Y are true, *then* we can be sure that conjecture Z is true.

Despite its intellectual honesty, I foresee some dire problems with this approach. Most alarmingly, it invites teachers who are pressed for time, or who perhaps are somewhat uncomfortable with mathematical proofs themselves, to stop after Chapter 12 and never get to a serious consideration of real deductive reasoning in geometry. In addition, whether students ever see Chapter 13 or not, the fact that *every* mathematical fact in the first 12 chapters is presented as a conjecture is likely to foster some very dangerous impressions: that mathematical truths are nothing but a bunch of conjectured generalities based on empirical experience, and that proofs are an afterthought that one might or might not choose to pursue once one has arrived at a convincing conjecture.

For these reasons, I would strongly discourage the District from choosing this book. It represents a highly risky and experimental approach to teaching geometry, and I think the experiment, while well-intentioned, is unlikely to have the desired effect.

College Preparatory Mathematics: Geometry Connections

This book is, in a word, unsuitable. To be sure, it presents a wealth of nice inquiry activities that might help students to develop an intuitive understanding of geometric relationships. But the problems with the book so outweigh its positive aspects that adopting it would lead to disaster.

Here are the main problems that I noticed with the book:

- The organization of chapters is obscure. Many of the chapters have titles that suggest two or more completely different ideas thrown together ("Justification and Similarity," "Proofs and Quadrilaterals"), and the chapters themselves did not give me any confidence that the authors had succeeded in integrating their disparate subjects into a coherent narrative.
- There are virtually no mathematically coherent definitions. One absolute requirement for deductive reasoning is precise definitions: You cannot argue carefully about a concept if you don't know exactly what it is. The descriptions that pass for "definitions" in this book are laughably vague, and many key concepts (such as *angle*) seem never to be defined at all.
- Many (perhaps most?) important geometric facts are never stated precisely. Instead, it is left to the students to glean a generality from their inquiry activities and then state it themselves. I don't dispute the usefulness of having students come up with their own versions of general statements based on guided experience; but it's a rare student who is able to synthesize experience into a correct and precise statement of a mathematical truth.
- There are almost no proofs in this book. For a while, the book guides students through what it calls "justifications" (which I think are supposed to be reasonably convincing arguments without quite having the logical force of proof), but it never says exactly what a "justification" is or how students can distinguish a good one from a bad one. Then in Chapter 7, the book introduces the notion of "proof," but never says exactly what a proof is or how it is supposed to be different from a "justification." Most damningly, I could not find a single proof presented clearly and completely in the book; instead, students are supposed to construct their own. How can they possibly know how to construct a good proof if they've never seen one?
- There are no postulates or axioms anywhere in the book, as far as I can tell. (There is no entry for either term in the index; and while there are glossary entries for both "axiom" and "proof," they don't have page references associated with them, unlike most of the other glossary terms.) This makes a mockery of the whole idea of proof, because a deductive proof has to be based on previously established results. If you try to prove anything without starting with some postulates, your argument will be either circular or too vague to be meaningful.

If this book is adopted, I shudder for the intellectual fate of a generation of Seattle high-school students.

Algebra Texts

I spent far less time with the algebra texts, so I won't have as much to say about them. Mainly, my overall impressions of the three algebra texts are parallel to my impressions of the geometry texts by the same publishers.

Prentice-Hall: Algebra 1 and Algebra 2

These books are solid, well organized, and mathematically sound. Computational algorithms and formulas are clearly stated and well motivated by examples and hands-on activities. The use of graphics to call out important principles is very effective.

Key Curriculum Press: Discovering Algebra and Discovering Advanced Algebra

My first impression is that these books have far too much verbiage for students to read, and too little in the way of clearly stated mathematical principles. Definitions, computational algorithms, and formulas seem to be stated vaguely when they are stated at all. For example, a *relation* is defined as "any relationship between two variables." Besides the fact that this is a circular definition, it is maddeningly vague – how is a student supposed to distinguish something that is a relation from something that is not?

College Preparatory Mathematics: Algebra Connections and Algebra 2

Like the geometry book from the same publisher, these books are completely unsuitable for a high-school algebra course. There are lots of nice hands-on activities here (algebra tiles, webs, generic rectangles, etc.), but very little in the way of clearly stated general principles. Many definitions of mathematical terms are utterly useless. For example, here are the definitions of relations and functions, two of the central concepts of algebra: "Each equation that relates inputs to outputs is called a *relation;* when a relation is functioning consistently and predictably, we call that relation a *function*." Again, both definitions are circular, and the key terms used in them are not clearly explained (except by example). Exactly what does it mean for a relation to "function consistently and predictably"?

Calculus Texts

I was only able to spend 15 minutes with the calculus texts, so I have very little to contribute. For what it's worth, I thought any of them could be adequate for a high school AP Calculus course. If pressed to rank them, I would put them in this order from best to worst:

1. Houghton Mifflin, *Calculus*
2. Pearson Prentice Hall, *Calculus: Graphical, Numerical, Algebraic*
3. Key Curriculum Press, *Calculus*

Sincerely,

John M. Lee
Professor of Mathematics

Cc: Members of the Seattle School Board